QUANTUM CHEMISTRY

HOMEWORK 8

Problems 1 and 2:
Sakurai Problems 17 and 23 of Chapter 3

Problem 3:
Prove Equation 3.6.11 of Sakurai [You can do this problem two ways, either by using the rotation operator (clever), or by grinding out the partial derivatives (longer, but possibly easier to work out)]

Recall that $L_i = \epsilon_{ijk} x_j p_k$

Problem 4 (From Baym's, *Lectures on Quantum Mechanics*):
(a)Find the energy levels and wave functions of a two-dimensional isotropic harmonic oscillator with the following potential:

$$V(r) = \frac{m\omega_\circ^2 r^2}{2} \; where \; r^2 = x^2 + y^2$$

by solving the Schrödinger equation in Cartesian coordinates. Find the degeneracy of each level. Write out the wave functions of the ground state, and each of the first excited states.
(b)Write the Schrödinger equation for this problem in polar coordinates*. Explicitly construct the wave functions of first excited states with angular momentum $+\hbar \; and -\hbar$; these wave functions are linear combinations of the wave functions found in part (a). (Hint: Bessel Functions)

*You can simply take the Laplacian in polar coordinates to be:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$